**Q1** Let A, B be subsets of R such that sup(A) and inf(B) exist and sup(A) < 0. Define

Explain why this makes sense (i.e. why for every ), and

prove that inf(C) exists.

By definition of supremum,

Thus,

Let For , we have ; thus, For we have

. Therefore,

Thus, is a lower bound for C, proving that C is bounded below. By the theorem derived from the completeness axiom, exists, as required.

**Q2** Show directly from the definition that

By the definition of convergence,

Let be arbitrary. For , we have

By the polynomial estimation lemma, there exist such that

Particularly, when we have

provided also satisfies Therefore, take with For with we have , so , as required.

**Q3**: For and , the quantity is defined to be the unique positive real numbers which has

a) For , use the binomial expansion for to show that and deduce that

By the binomial theorem and for such that we have

When . For any because all subsequent terms in the sum are positive for naturals and Thus,

Since both sides of the inequality are positive,

Furthermore,

Again, they can be exponentiated to make

Combining both inequalities gives us

as required.

b) Use the previous part to show, directly from the definition, that

Let be arbitrary. For , we have

(the absolute value does not change anything since the LHS is non-negative).

Since (from Q3 part a),

Furthermore,

provided satisfies . From combining the two inequalities, we get

Take such that . Then for all That is, as , as required.